Higher Maths KO

Algebra KS4 Part 1 H

Solving linear equations

$6x - 5 = 2x + 15$

$-2x \qquad -2x$

Subtract 2x from both sides

$4x - 5 = 15$

$+5 \quad +5$

Add 5 to both sides

$4x = 20$

Divide both sides by 4

$x = 5$

Factorising

Factorise $x^2 + x - 42$

product of - 42

$-6 \times 7 = -42$

Sum of (+) 1

$-6 + 7 = 1$

$x^2 + x - 42 = (x - 6)(x + 7)$

Expanding Brackets

$(5x - 3)(2x + 1)$

×	5x	- 3
2x	10x²	- 6x
+1	+5x	-3

$= 10x^2 - 6x + 5x - 3$

$= 10x^2 - x - 3$

Don't forget to simplify
$-6x + 5x = -x$

Rearranging formulae

a

$y = ax + c$

$ax + c = y$

$ax = y - c$

$x = \frac{y-c}{a}$

The 'aim' is to get 'x' on its own (x = ...)

b

$ax - b = cx + b$

$ax - cx = 2b$

$x(a - c) = 2b$

$x = \frac{2b}{a-c}$

Factorise to isolate the 'x'
$ax - cx = x(a - c)$

Rearrange so that all the terms involving 'x' are on the same side

Divide by (a - c) to leave 'x' on it's own

Algebra KS4 Part 2 H

Inequalities

(a)

$7x + 3 < 2x + 18$

$5x + 3 < 18$ (-2x)

$5x < 15$ (-3)

$x < 3$ (+5)

$-1 < 2x + 5 < 9$
-5 -5 -5

(b)

$-6 < 2x < 4$

$-3 < x < 2$ Integer solutions : -2, -1, 0, 1

Sequences

Find the nth term: 15, 10, 5, 0

Look at the difference between consecutive terms - decreasing by 5 each time

So we know the nth term formula will include **-5n**

-5n   -5   -10   -15   -20

     +20   +20   +20   +20

Sequence   15   10   5   0

What have we added to each number to get from the top line to the sequence?

The nth term = -5n + 20 which can be written 20 - 5n

SIMULTANEOUS EQUATIONS

Solve simultaneously

$3x - 2y = 8$ and $2x - 3y = 7$

To make the 'y's match multiply the first equation by 2 and the second equation by 3

$9x - 6y = 24$

$\underline{4x - 6y = 14}$

$5x + 0 = 10$

-6y - -6y = 0 so we will subtract (Same signs -)

$x = 2$ $6 - 2y = 8$

         $y = -1$

We could have solved by matching the 'x' by multiplying the first equation by 2 and the second equation by 3

Using the quadratic formula

$x^2 - 3x - 1 = 0$

$a=1 \quad b=-3 \quad c=-1$

$b^2 - 4ac$

$= (-3)^2 - 4 \times 1 \times -1$

$= 13$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\frac{3 \pm \sqrt{13}}{2}$ $x = 3.30 \quad x = -0.30$

Calculation KS4 Part 1 H

Ratio

Divide £48 in the ratio 3 : 5

3 : 5   8 parts in the ratio (3 + 5)
£48 ÷ 8 = £6   1 part = £6

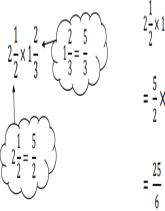3 parts = 3 × £6 = £18
5 parts = 5 × £6 = £30

£18, £30

INDICES
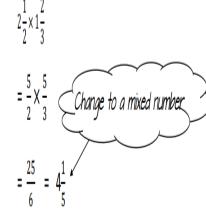
Remember $a^{-n} = \frac{1}{a^n}$

$3^{-1} = \frac{1}{3}$ $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^{2} = \frac{9}{4} = 2\frac{1}{4}$

Fractions

$\frac{1}{2} \times 1\frac{2}{3}$ $\frac{2}{1} = \frac{5}{3}$
$2\frac{1}{2} \times 1\frac{2}{3}$ $1\frac{1}{3} = \frac{5}{3}$

$\frac{1}{2} \frac{5}{2}$
$2\frac{1}{2} = \frac{5}{2}$

$= \frac{5}{2} \times \frac{5}{3}$ *Change to a mixed number*

$= \frac{25}{6} = 4\frac{1}{5}$

Prime Factor Form

Write 72 as a product of its prime factors.

72
② 36
② 18
② 9
③ ③

2 × 2 × 2 × 3 × 3

ok $2^3 × 3^2$

$2^3 × 3^2$

Standard Form

Work out $6 × 10^3 × 5 × 10^2$

$6 × 10^3 × 5 × 10^2$

$= 6 × 5 × 10^3 × 10^2$

$= 30 × 10^5$ *$10^a × 10^b = 10^{a+b}$*

$= 3 × 10^6$

Make sure your answer is in standard form

Calculation KS4 Part 2 H

Counting Rule

Mr Idris has 5 pairs of trousers, 9 shirts and 3 ties.

Work out the total number of ways of choosing a pair of trousers, a shirt and a tie.

$$5 \times 9 \times 3$$

$$\underline{135}$$

INDICES

HINT : Deal with the negative first

$$36^{-\frac{1}{2}} = \left(\frac{1}{36}\right)^{\frac{1}{2}}$$
$$= \frac{1}{6}$$

$$\left(\frac{4}{9}\right)^{-\frac{3}{2}} = \left(\frac{9}{4}\right)^{\frac{3}{2}}$$
$$= \frac{27}{8}$$
$$= 3\frac{3}{8}$$

SURDS

Simplify $3\sqrt{7} + 2\sqrt{7}$

$\sqrt{7} + \sqrt{7} + \sqrt{7}$ + $\sqrt{7} + \sqrt{7}$

$= 5\sqrt{7}$

Bounds

A rectangle has a length of 21cm, to the nearest cm, and a width of 5.3cm, to the nearest mm.

(a) Work out the upper bound for the perimeter of the rectangle.

20 ↑ 21 ↑ 22   5.2 ↑ 5.3 ↑ 5.4
  20.5 21.5   5.25 5.35

$$2(21.5) + 2(5.35)$$

$$\underline{53.7}$$

Simplify $\sqrt{24} \times \sqrt{27}$

$$\sqrt{24} = \sqrt{4} \times \sqrt{6} = 2\sqrt{6}$$
$$\sqrt{27} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$$

$2\sqrt{6} \times 3\sqrt{3} = 6\sqrt{18}$

$= 6 \times \sqrt{9} \times \sqrt{2}$
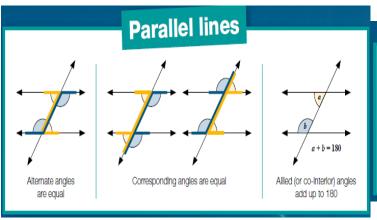
$= 18\sqrt{2}$

Calculation – Key Words

- Integer - a whole Number
- Fraction – a part of a number has a numerator on the top and a denominator on the bottom
- Equivalent Fraction – two fractions which have the same value but are written differently- ½ = ⁴/₈
- Percent – means out of 100, symbol %
- Multiple – any number in your original times table
- Factor - a number that goes into another number with no remainder
- Highest Common Factor – the biggest number that goes into two numbers – HCF of 12 and 16 is 4
- Lowest Common Multiple – the first number that appears in the times table of 2 different numbers - LCM of 3 and 5 is 15
- Prime Number - a number with only 2 factors, itself and 1
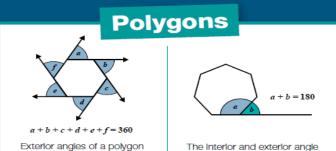- Square number – the answer to a number multiplied by itself
  1 x 1 = 1   2 x 2 = 4   3 x 3 = 9
- Cube Number - the answer to a number multiplied by itself twice
  1 x 1 x 1 = 1   2 x 2 x 2 = 8   3 x 3 x 3 = 27
- Square Root - Opposite of square number. This is the answer to what number multiplied by itself is the square number - √16 = 4 x 4 so square root of 16 is 4
- Product means to multiply
- Sum means to add
- Share means to divide
- Difference means to subtract
- Evaluate – work out the answer
- Ratio – is comparing one quantity against another, written as a: b
- Significant figures – Is rounding to the most important (biggest value) digit – 2567 to 1 sf is 3000
- Standard Form – Is a method of writing very large or very small numbers – a x 10ⁿ. Where a is bigger than 1 and smaller than 10
- BIDMAS – Gives the order in which a calculation should be done
- Recurring – A decimal that continues forever with the same number after the decimal point
- Prime Factor – A factor of a number that is also a Prime number.
- Mixed Fraction - A fraction that has a whole number and a fraction
- Top heavy Fraction – A fraction where the numerator is bigger than the denominator
- Indices – are another name for powers. There are rules with indices
- Surd – is a square root of a number that does not give an integer - √11

Geometry KS4 Part 1 H

Parallel lines

Alternate angles are equal

Corresponding angles are equal

Allied (or co-interior) angles add up to 180

$a + b = 180$

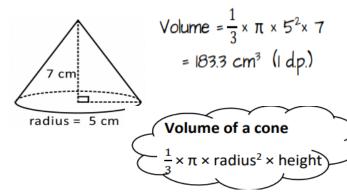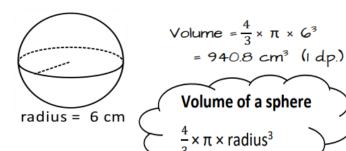## Polygons

$a + b + c + d + e + f = 360$

Exterior angles of a polygon add up to 360

$a + b = 180$

The interior and exterior angle of any polygon add up to 180
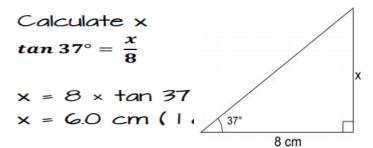
Geometry Key Words and Formula's

* Area – the space inside a 2D shape measured in units 2
* Perimeter - the distance around the outside of a shape (called circumference for circles
* Volume – the space inside a 3D shape
* Surface Area – the area of the flat faces of a 3D shape
* Angle - The space made when two lines meet, measured in degrees
* Acute angle – less than 90^0, obtuse angle bigger than 90 smaller than 180^0. Straight line angle equal to 180^0. Reflex angle bigger than 180 but smaller than 360^0.
* Angles in a straight line add to 180^0
* Angles in a triangle add to 180^0
* Angles around a point add to 360^0
* Parallel lines – these lines have the same gradient and they never meet
* Perpendicular lines – these lines cross at 90^0
* Alternate angles – these two angles are the same in parallel lines (Z angle)
* Corresponding angles – these two angles are the same in parallel lines (F angles)
* Co – Interior angles – these two angles add up to 180^0 (C angles)
* Scalene triangle – A triangle with three different sides and three different angles
* Isosceles triangles – A triangle that has the two sides the same length and the base angles the same
* Equilateral triangle – A triangle that has three sides the same and three angles the same
* Polygon - A 2D shape that has only straight sides (edges)
* Interior and Exterior angles – The exterior angles of any polygon always add to 360^0. The interior angles + exterior angles always add to 180^0
* Bearing - A bearing is an angle that starts from North, goes clockwise and must have three figures in it
* Reflection – is flipping a shape over a mirror line
* Rotation – Is spinning a shape from a certain point, the direction of movement is required
* Translation – is pushing a shape horizontally and vertically. It is written as a vector
* Enlargement – is making a shape bigger or smaller by a scale factor from a centre point
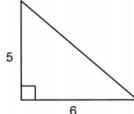
Volumes

Cuboid = $l \times w \times h$	
Prism = area of cross section × length	
Cylinder = $\pi r^2 h$	
Volume of pyramid = $\frac{1}{3} \times$ area of base $\times h$	

Calculate the volume

7 cm

radius = 5 cm

$$\text{Volume} = \frac{1}{3} \times \pi \times 5^2 \times 7$$
$$= 183.3 \text{ cm}^3 \ (1 \text{ d.p.})$$

Volume of a cone

$$\frac{1}{3} \times \pi \times radius^2 \times height$$

Calculate the volume

radius = 6 cm

$$\text{Volume} = \frac{4}{3} \times \pi \times 6^3$$
$$= 940.8 \text{ cm}^3 \ (1 \text{ d.p.})$$

Volume of a sphere

$$\frac{4}{3} \times \pi \times radius^3$$

Geometry KS4 Part 2 H

Calculate x

$\tan 37° = \dfrac{x}{8}$

$x = 8 \times \tan 37$

$x = 6.0 \text{ cm } (1 \cdot$

37°

8 cm

Calculate the length of the missing side

$5^2 + 6^2 = x^2$

$x^2 = 61$

$x = 7.8 \text{ cm } (1 \text{ d.p.})$

5

6

X

Trigonometric formulae

Sine Rule	$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule	$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle	$= \dfrac{1}{2} ab \sin C$

C

b   a

A   c   B

Sine Rule

5 m

42°

x

53°

A

B

C

Work out the length of BC.
Give your answer to 3 significant figures.

$\dfrac{x}{\sin(42)} = \dfrac{5}{\sin(53)}$

$x = \dfrac{5}{\sin(53)} \times \sin(42)$

$= 4.19 \text{ (3sf)}$

......4.19.....

Circle Theorems

Pythagoras

Pythagoras' Theorem

For a right-angled triangle,
$a^2 + b^2 = c^2$

c   b

a

Trigonometric ratios (*new to F*)

$\sin x° = \dfrac{\text{opp}}{\text{hyp}}, \cos x° = \dfrac{\text{adj}}{\text{hyp}}, \tan x° = \dfrac{\text{opp}}{\text{adj}}$

hyp   opp

x°

adj
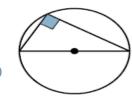
The angle at the centre of a circle is twice the angle at the circumference

x

2x

The angle in a semicircle is a right angle (or 90°)

Alternate segment theorem

Angles in the same segment are equal

x   x

y   y

Opposite angles in a cyclic quadrilateral add to 180

a

d   b

c

$a + c = 180$
$b + d = 180$

A tangent to a circle is perpendicular (or 90°) to the radius.

A

B

C

$CA = CB$

Tangents to a circle from an external point are equal in length
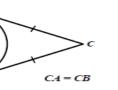
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* Enlargement – is making a shape bigger or smaller by a scale factor from a centre point
* Congruent Shapes – Are shapes which are the same - they have the same angles and the same sides
* To prove shapes are Congruent use SAS, SSS, ASA or RHS
* Similar Shapes – Have the same angles but the sides are in ratio. Find the scale factor of the big shape to the smaller one
* Pythagoras – Is used for right angled triangles to find the length of a missing side. Square both, add or subtract then square root
* SOH/CAH/TOA – is used to find either a missing side or a missing angle in a right-angled triangle
* hypotenuse – is the longest side in a right-angled triangle
* O, A H - are the opposite, adjacent and hypotenuse. Used with SOH/CAH/TOA
* Cosine Rule – To find and angle or a side of any triangle. Use when only one angle in the question
* Sine Rule – Used to find a side or an angle in any triangle. Use when two pairs of sides and angles are involved in the question.
* Tangent – Is a line that touches a curve once only in a circle a tangent meets a radius at 90°
* Chord – Is a line that goes from one end of the circle to the other but not through the centre
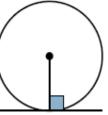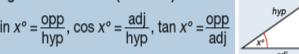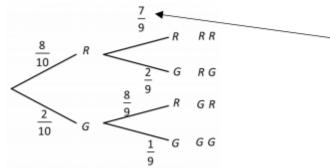* Segment – Is part of a circle that is made up by a chord.

Statistics KS4 Part 1 H

Probability

A bag contains 8 red and 2 blue balls. Two balls are selected at random

Take care with the probabilities for the second pick - now 9 balls left in the bag. If a red picked out first then there are only 7 red balls left for the second pick.

$\frac{8}{10}$ R

$\frac{7}{9}$ R   R R

$\frac{2}{9}$ G   R G

$\frac{8}{9}$ R   G R

$\frac{2}{10}$ G

$\frac{1}{9}$ G   G G

Venn Diagram

Sami asked 50 people which drinks they liked from tea, coffee and milk.

All 50 people like at least one of the drinks
19 people like all three drinks.
16 people like tea and coffee but do not like milk.
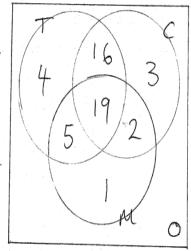21 people like coffee and milk.
24 people like tea and milk.
40 people like coffee.
1 person likes only milk.

Sami selects at random one of the 50 people.

Work out the probability that this person likes tea.

$\frac{44}{50}$

Mean from a table

Adam is measuring the heights in cm of his tomato plants.

Height (cm)	m.p	Frequency	m.p × f
$140 < h \leqslant 150$	145 ×	7	1015
$150 < h \leqslant 160$	155 ×	10	1550
$160 < h \leqslant 170$	165 ×	15	2475
$170 < h \leqslant 180$	175 ×	19	3325
$180 < h \leqslant 200$	190 ×	9	1710
		60	10075

(a) Estimate the mean height.
Give your answer correct to 1 decimal place.

$\frac{10075}{60} = 167.9 \ (1dp)$

Statistics KS4 Part 2 H

Cumulative Frequency

The frequency table shows the time taken for 100 people to travel to an event.

Time (minutes)	Frequency	C.F
$20 < t \leqslant 30$	9	9
$30 < t \leqslant 40$	16	25
$40 < t \leqslant 50$	20	45
$50 < t \leqslant 60$	29	74
$60 < t \leqslant 70$	15	89
$70 < t \leqslant 80$	..	100

(a) On the grid, plot a cumulative frequency graph for this information.

(b) Find an estimate for the median time taken.

52 minutes

Box Plots

The times, in seconds, of 15 students running a race are recorded below.

52   54   54   55   58   58   59   (60)   60   61   61   64   67   70   75

Draw a box plot for this information.

median = 60
LQ = 55
UQ = 64

The table shows information about the age of 80 teachers.

F.d = Freq / width

Age (years)	Frequency	F.d.	
$20 < a \leqslant 30$	20	2	[20 ÷ 10]
$30 < a \leqslant 35$	22	4.4	[22 ÷ 5]
$35 < a \leqslant 40$	16	3.2	[16 ÷ 5]
$40 < a \leqslant 50$	13	1.3	[13 ÷ 10]
$50 < a \leqslant 65$	9	0.6	[9 ÷ 15]

On the grid, draw a histogram for the information in the table.

Age (Years)

Statistics Key Words

Range – the difference between the largest and the smallest number from a list of numbers

Mode – The number that appears the most often from a list of numbers

Median – the number in the middle of a list of ordered numbers

Mean – Add all the numbers up and then divide this total by the amount of numbers that were there

Averages – Mean, Median and Mode. Three averages to help determine common or a representative number from a list of numbers

Pie Charts – A way of representing data in a circle. All pie charts add up to 3600.

Probability - the chance of an event happening. Probability must be written as a fraction, decimal or a percentage. Not as a ratio.

Tree Diagrams – A diagram to show the probability of two or more events happening

Probability 'OR' Rule – The Probability of event A or event B happening is P(A) + p(B)

Probability 'AND' Rule – The Probability of event A and event happening is P(A) x P(B)

Probability – of all possible events adds to 1

Venn Diagrams – Uses two circles often overlapping to show data

Scatter Diagram – A graph that shows the relationship between two variables

Correlation – Used to describe the relationship in scatter diagrams – positive both go up or down, negative – one goes up as the other goes down, no correlation – there is no link between the two variables

Line of best Fit – A straight line drawn through the scatter diagram with roughly half the data points on either side of the line

Stem and Leaf – Shows numbers in a table, the leaf is the last digit of the piece of data. Needs a key

Histogram – A diagram that uses the area of a section to represent the frequency, not the height. It looks like a bar graph with bars of different widths

Cumulative Frequency - A diagram that can be used to find the median from the curve on the graph. Cumulative frequency, means the frequency added up

Box Plots – Uses a box which contains the middle 50% of the data. Diagram has Lowest Value, Lower Quartile, Median, Upper Quartile and Highest Value

Lower and Upper Quartiles- these are 25% and 75% of the way through the ordered data

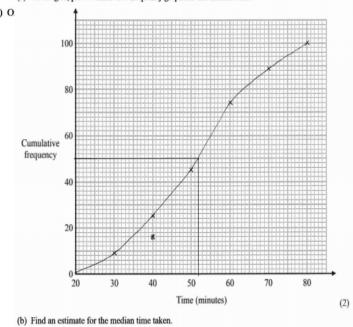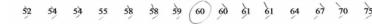Inter-Quartile Range – Upper Quartile – Lower Quartile. It is the middle 50% of the data